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## ABSTRACT

This report is a continuation of a study conducted at the University of Wisconsin during the spring of 1967. The previous study, Technical Report #38, succeeded in teaching conservation of numerosness to small groups of kindergarten children, in a middle-class community. The purpose of the present study was to determine if the typical classroom teacher, in schools differing in socio-economic levels, could successfully use the lessons developed in the previous study to effect conservation of numerosness with kindergarten children. Four questions were considered - (1) can the typical classroom teacher teach the conservation lessons as successfully as a specially trained expert, (2) is the treatment of greater value for pupils from disadvantaged backgrounds, (3) is the treatment of greater value for younger kindergarten children than for older ones, and (4) do younger children from disadvantaged backgrounds, who may have more cognitive flexibility, benefit more from the lessons? The "Test of Conservation of Numerousness" was presented to 484 kindergarten students partitioned into a control and two experimental groups. The results showed that the only question which could be answered affirmatively was question number two. (RP)

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FINAL REPORT

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THE IDENTIFICATION OF SOCIO-ECONOMIC DIFFERENCES AND THEIR EFFECT ON  
THE TEACHING OF READINESS FOR "NEW MATH CONCEPTS" IN THE KINDERGARTEN

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April 1970

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# The Identification of Socio-Economic Differences and Their Effect on the Teaching of Readiness for "New Math Concepts" in the Kindergarten

## I. INTRODUCTION

This project is a continuation of a study conducted under the sponsorship of the Wisconsin Research and Development Center for Cognitive Learning, University of Wisconsin, Madison, Wisconsin during the Spring of 1967. The Study is reported in Technical Report No. 38. The previous study succeeded in teaching conservation of numerosness to small groups of kindergarten children, in a middle-class community, with one highly trained teacher conducting the lessons. The purpose of the present study was to see if the typical classroom teacher, in schools differing in socio-economic levels and with whom the ultimate value of the treatment rests, can successfully use the lessons developed in the previous study to effect conservation of numerosness with kindergarten children. An additional purpose was to ascertain whether special in-service training is necessary for the treatment to have its full educational impact on the pupils.

The 1967 Study in Oconomowoc, Wisconsin Public Schools was successful, but this community does not have many children from low socio-economic families. In a pilot study, using the same lessons in Racine, Wisconsin, the lessons were used with disadvantaged children, and they appeared to be somewhat less successful than in Oconomowoc. For this reason, the investigator felt it necessary to use the lessons in a community where adequate samples of all socio-economic levels could be used. Many authorities frequently contend that children from low socio-economic levels are particularly in need of this kind of training (3).

The Study of 1967 controlled the teacher variable by having only one teacher (who received instruction and demonstrations for teaching each lesson) conduct the experimental lessons. To make this study more generalizable, teachers were chosen at random and some given training on how to teach the lessons while others proceeded on their own. Also, inexpensive, readily obtained materials were used in the 1967 experiment, and these were consistent for all classes. In the present study we proposed to let each teacher use those materials he could obtain most easily. The same materials used in the previous study were specified in the teachers' guide to the lessons but the investigator suggested that they might use alternative materials as long as the general procedure and format of the lessons was not violated.

By allowing for these variations, the applicability of the lessons to "normal" classroom situations allowed a better analysis of the desirability of using the lessons with all kindergarten children under the typical teacher's direction. The following questions were considered:

1. Can the typical classroom teacher teach the conservation lessons as successfully as a specially trained expert? Is special training necessary or beneficial for the treatment to have maximum benefit?

2. Is the treatment of greater value for pupils from disadvantaged backgrounds? (Is there a treatment by socio-economic-status interaction?)

3. Is the treatment of greater value for younger kindergarten children than for older ones? If environmental deprivation has a crystallizing effect, as is often claimed, one might expect to find this to be the case. (Is there a treatment-by-age interaction?)

4. Do younger children from disadvantaged backgrounds, who may have more cognitive flexibility, benefit more from the lessons? (Is there a treatment-by-socio-economic-status-by-age-interaction?)

The concepts espoused by Jean Piaget have received acclaim by some and criticism by others. His study of conservation as it relates to mathematical thinking has been the subject of many research studies. The present study is another in the general area of the children's abilities to recognize numerical properties of sets of objects.

A concern of this study is not to introduce something new to the kindergarten curriculum but rather to present a means of introducing children to an important mathematical concept in a limited number of relaxed and enjoyable teaching learning situations. It is the investigator's opinion that a reduction in formal (often frustrating) experiences for kindergarten children might therefore supplant some of the published materials now used on a daily basis with children at this level.

Conservation of numerosness as it was used in this study can best be explained by an example. Place before a pre-school child two rows of counters that are paired and ask the question,

0 0 0 0 0

0 0 0 0 0

"Are there the same number of counters in both rows or does one have more than the other?" He will most likely respond that they both have the same number because (1) He can establish a one-to-one correspondence between the two sets or (2) He will respond to the length of the two rows.

Next, change one row by spreading the counters over a larger area and ask the same question.

0 0 0 0 0

0            0            0            0            0



If the child conserves numerosness, he will respond that there are the same number of counters in both rows. Non-conservers will indicate that the "longer" row has more counters if they base their judgments on the length of the rows. In other words, the conservers of numerosness realize that changing the form in which the counters are displayed does not affect their cardinality.

Many authorities feel that conservation of numerosness is basic to success in number experiences. Steffe's study (38, p. 31) seems to place further emphasis on the value of children possessing this conservation ability. He found that first grade children who were conservers of numerosness were better problem solvers than the non-conservers.

Several studies have been conducted to see whether conservation of numerosness can be enhanced through teaching. Piaget has not been altogether clear on his stand on this issue. At one time he indicated that teaching could, no doubt, have an important effect on a child's ability to conserve number. Piaget has stated (32, p. 1) that the development of the intellectual capacity of children depends on at least the following factors: (1) maturation, (2) encounters with experience, (3) social transmission (teaching, etc.), and (4) equilibrium which has been described as ". . . the mental activity of the subject when confronted with cognitive conflict. . ." (36, p. 325).

Duckworth interprets Piaget's stand on teaching as follows:

Good pedagogy must involve presenting the child with situations in which he himself experiments, in the broadest sense of that term -- trying things out to see what happens, manipulating things, manipulating symbols, posing questions and seeking his own answers, reconciling what he finds one time with what he finds at another, comparing his findings with those of other children (32, p. 2).

Adler also emphasizes the importance of environmental stimulation:

Piaget's critics have often complained that his emphasis on inward maturation and growth leaves no room for the effects of a stimulating environment. This view involves a partial misunderstanding of his theory, and the difficulty could be resolved easily by the realization that Piaget assumes continuous interaction between the child and his environment (2, p. 300).

Every normal pupil is capable of sound mathematical reasoning if his own initiative is brought into play.

The real cause of the failure of formal education must be sought primarily in the fact that it begins with language (accompanied by illustrations and fictitious or narrated action) instead of beginning with real practical action. The preparation for subsequent mathematical teaching should begin in the home by a series of

manipulations involving logical and numerical relationships, the idea of length, area, etc., and this kind of practical activity should be developed and amplified in a systematic fashion throughout the whole course of primary education. . . (2, p. 301).

The order in which a child progresses through the four major stages of mental growth is fixed, but his rate of progress is not fixed. The transition from one stage to the next can be hastened by enriched experience and good teaching. Carpenter quotes Piaget's expression of this idea from The Growth of Logical Thinking from Childhood to Adolescence:

The maturation of the nervous system can do no more than determine the totality of possibilities and impossibilities at a given stage. A particular social environment remains indispensable for the realization of these possibilities. It follows that their realization can be accelerated or retarded as a function of cultural and educational conditions (33, p. 74).

There are other occasions, however, where his statements make one wonder whether he really believes that the acquisition of conservation can be effected at all through teaching.

It is a great mistake to suppose that a child acquired the notion of number and other mathematical concepts from teaching. On the contrary, to a remarkable degree he develops them himself, independently and spontaneously. When adults try to impose mathematical concepts on a child prematurely, his learning is merely verbal; true understanding of them comes only with his mental growth (33, p. 74).

"Children go through certain stages of intellectual development from birth through adolescence. These stages materialize, fully constructed, when their time has come, and there is little we can do to advance them" (35, p. 1). The above quotation is Eleanor Duckworth's interpretation of Piaget.

Piaget is very elusive with regard to his personal convictions on the matter of enhancing learning of conservation at early ages. However, there is no question concerning his beliefs about the importance of children being able to conserve both number and substance. He states, ". . . children must grasp the principle of conservation of quantity before they can develop the concept of number."

Some investigators and writers express their concern for the differences in the learning rates of children from differing socio-economic levels in various content areas. Dutton (17, pp. 358ff.) found that children from culturally disadvantaged areas had difficulty in learning to tell time. It was his belief that this resulted from the lack of emphasis on orderliness and sequence in their home environments.

The influence of the home and the responsibility of the schools is further emphasized by Farnham-Diggory when she states that:

. . . 'readiness' is not a simple, chronological function. The growing human constructs strategies for dealing with the world, and these strategies change with development and experience. Pedagogy at any age should take these natural coping systems into consideration and design instructional schemes to fit and improve them. It can be dangerous and wasteful to wait until a child haphazardly develops a particular skill (as defined by standard mathematical readiness tests, for example) before offering him instruction.

First of all, unless he comes from a high favored environment, the child may never develop such skills by himself. There is a growing body of research showing that early isolation from systematic learning experiences -- whether this isolation results from the randomness of a noisy slum, or the desolate regimentation of a bad orphanage -- may produce irreversible defects in cognitive development (25, p. 620).

Though we have not tried to make a study of racial implications in the learning of conservation of numerosness it is commonly proclaimed that certain ethnic groups do not have effective learning experiences because of the lack of exposure to various ethnic groups.

One kindergarten class in our study was composed of all Negro children except for one and that child was of Mexican descent. The classrooms varied in degree of integration from the case above to those composed of entirely Caucasian parentage.

Atkinson (4, pp. 241-251) found that Negro children had poor self-images which affected their ability to learn.

Negroes were told and believed that they were inferior and would fail and therefore they failed. Thus for the black child to be motivated to achieve in school, the school, must negate everything that the society affirms: it must tell the child that he can succeed and then he will succeed (4, p. 250).

## II. Methods

The "Test of Conservation of Numerosness" (27, pp. 16-18) used in the 1967 study was designed to be used with small groups. A Hoyt-Reliability coefficient of .91 was computed with an item analysis for the instrument. It was also correlated with an individually administered test and a correlation between total scores of .84 was obtained. The test functioned well in the previous study but the investigator felt that the item analysis indicated a need for revising some of the instructions and physical arrangements of figures on a few of the items. For this reason, a revised form was developed and was correlated with the original instrument.

A random sample of kindergarten children was employed to test the reliability of Form III (the present revision) of the "Test of Conservation of Numerousness." Two schools were selected in Longmont, Colorado for this purpose. The schools selected were chosen because of their wide range of socio-economic status classes in their attendance districts. Form III of the test was administered in early December, 1968. Form I was then administered after Christmas vacation in late January to the same children for purposes of comparing responses.

The correlation coefficient between the total scores of the test was .70. Though this was not as high as was obtained on the earlier form of the test. The present form (Form III) was employed in this study because its physical arrangement made it easier to administer to the kindergarten children. (There was no need for the children to turn the booklets as in the earlier form.) An item analysis also revealed that the test was discriminating well on items of differing types.

Included below are the sample pages and instructions for Form I of the test used in Oconomowoc.

W-1 Look at the squares on both pages. Are there the same number of squares on both pages? Or does one page have more than the other? Show me by pointing. Don't talk out loud. If you think both pages have the same number of squares, put a finger on both pages. (Make sure the children are using both hands.) If you think one page has more squares on it, put your finger on that page. Don't take it away until I tell you. Turn your book to the pages with the bee at the top.

W-2 Look at the squares on both pages. Remember what you are suppose to do with your hands. Listen carefully. Are there the same number of squares on both pages? Or does one page have more than the other? Show me by pointing. Turn to the page with the car on the top.

W-3 (Three discs.)\* Put the discs on the squares. Notice there are the same number of discs as squares. Now move the discs to cover the dots. Are there the same number of discs as squares? Or are there more of one than the other? Show me by pointing. (Make sure they point with both hands or one depending on whether they think they are the same, etc.) Turn to the page with the tricycle at the top.

W-4 (One disc.) Place the disc on the dot. Are there the same number of squares as discs? Or are there more of one than the other? Show me. Turn to the page with the butterfly at the top.

\* Number of discs in parentheses indicate the amount the examiner is to give each child.



1. Look at the squares on both pages. Are there the same number of squares on both pages? Or does one page have more than the other? Show me. Turn to the page with the teddy bear at the top.
2. (Six Discs.) Put the discs on the squares. Now cover the dots with the discs. Are there the same number of discs as squares? Or are there more of one than the other? Show me. Turn to the page with the fish at the top.
3. (Five discs.) Put the discs on the squares. Now cover the dots with the discs. Are there the same number of squares as discs? Or are there more of one than the other? Show me. Turn to the page with the duck at the top.
4. (Five discs.) Put the discs on the squares. Move the discs to cover the dots. Are there the same number of squares as discs? Or are there more of one than the other? Show me. Turn to the page with the horse at the top.
5. Look at the squares on both pages. Are there the same number of squares on both pages? Or does one page have more than the other? Show me. Turn to the page with the sheep at the top.
6. (Five discs.) Put the discs on the squares. Move the discs to cover the dots. Are there the same number of discs as squares? Or are there more of one than the other. Show me. Turn to the page with the bear at the top.
7. Look at the dots in both rows. Are there the same number of dots in both rows? Or does one row have more than the other? Show me. Turn to the page with the turtle at the top.
8. Look at the squares on both pages. Are there the same number of squares on both pages? Or does one page have more than the other? Show me. Turn to the page with the chicken at the top.
9. (Seven discs.) Put the discs on the squares. Move some of the discs to cover the dots. Are there the same number of discs as squares? Or are there more of one than the other? Show me. Turn to the page with the tractor at the top.
10. (Six discs.) Cover each dot with a disc. Are there the same number of squares as discs? Or are there more of one than the other? Show me. Turn to the page with the chicken at the top.
14. Look at the dots on both pages. Are there the same number of dots on both pages? Or does one page have more than the other? Show me. Turn to the page with the clown at the top.
15. Look at the dots in both rows. Are there the same number of dots in both rows? Or does one row have more than the other? Show me. Turn to the page with the owl at the top.

16. (Six discs.) Put the discs on the squares. Move the discs to cover the dots. Are there the same number of squares as discs? Or are there more of one than the other? Show me.

All of the above instructions were used in Form III with the exception of item nine which was changed to read as follows:

9. (Seven discs.) Put the discs on the squares. Move some of the discs to cover the dots. Think about all the squares and all of the discs that were on the page. Are there the same number of squares as discs or is there more of one than the other? Show me. Turn to the page with the tractor at the top.

For the test format, see Appendix 1.

A Special Study Committee of the Denver Public Schools has completed an investigation of equality of educational opportunity in the district and has amassed much data on socio-economic levels in various sections of the school district. (41, pp. 9-12 append.)

Using the results of this study, a random, selection of eighteen classrooms was made in areas 1-6 as defined in the Special Study Committee publication and indicated on the map of Denver (appendix 2). The three classrooms per area are described below:

A. Control:

One control classroom which was tested at the end of the experiment. Nothing else was done with this classroom.

B. Experimental 1:

One experimental classroom where the teacher received special training by the principal investigator in how to teach each of the twelve lessons.

C. Experimental 2:

One experimental classroom where the teacher used his own interpretation of "Lessons for Teaching Conservation of Numerousness to Kindergarten Children."

The series of twelve weekly thirty-minute lessons (27, pp. 33-43) developed for the Oconomowoc experiment was duplicated and supplied to all of the experimental teachers. The control teachers were not informed that their classes were participating in an experiment.

The teachers in Experimental 2 above were merely brought together before the experiment began to give them the teacher's guides for the conservation lessons and a few general directions concerning materials and days for teaching lessons.

The investigator brought together the Experimental 1 teachers who were selected for the directed teaching. They met for approximately

one hour each week to discuss presentation of the lesson for the following week.

The investigator requested that all experimental lessons be taught on Wednesdays with makeups on Tuesdays or Thursdays.

The experiment was conducted during the spring semester, 1969. The test was administered to the children in both experimental and control classrooms using the revised Form III of the "Test of Conservation of Numerousness." This testing commenced one week after the close of the experiment.

For purposes of testing, the investigator trained three certificated primary teachers to conduct the testing. The children were tested in groups of five over a period of two weeks following the close of the lessons. Because the control children had not been involved with the lessons, they were tested during the week immediately following the experiment. The experimental groups were then tested one week after their last experimental lessons. Schools were tested on a random selection basis.

The subjects were classified according to their "head of family's" occupation. The categories presently employed by the Bureau of the Census were used to categorize occupations. This is a revision of Edwards List with some slight modifications. The complete list employed appears in Bonjean, *et al.* (9, pp. 424-429). The List includes six major categories. For purposes of this study the seventh category was added by the author because of the sizeable number of such subjects involved in the study. The major categories employed are as follows:

1. Professional, technical and kindred workers
2. Managers, officials and proprietors
  - A. Nonfarm managers, officials, and proprietors
  - B. Farmers and farm managers
3. Clerical and sales workers
4. Craftsmen, foremen, and kindred workers
5. Operatives and kindred
6. Unskilled, service, and domestic workers
  - A. Private household workers
  - B. Service workers, except private household
  - C. Farm laborers and foremen
  - D. Laborers, except farm and mine (9, p. 424)
7. Unemployed and "Aid to Dependent Children," etc.

Following is the schedule of activities performed during the course of the experiment:

1. Establish reliability of revised form of "Test of Conservation of Numerousness."
2. Random selection of eighteen classrooms for study in areas 1-6 as defined in Special Study Committee publication. (Three classrooms per area.)

### 3. Experiment begins

Jan. 28-30	A. Lesson I
Feb. 4-6	B. Lesson II
Feb. 11-13	C. Lesson III
Feb. 18-20	D. Lesson IV
Feb. 25-27	E. Lesson V
Mar. 4-6	F. Lesson VI
Mar. 11-13	G. Lesson VII
Mar. 18-20	H. Lesson VIII
Mar. 25-27	I. Lesson IX
Apr. 1-3	J. Lesson X
Apr. 8-10	K. Lesson XI
Apr. 15-17	L. Lesson XII

### 4. Data was collected on teachers and subjects while experiment was in progress.

#### A. Teachers' data

1. Age
2. Sex
3. Years of experience
4. Level of college training

#### B. Children's data

1. Age
2. Socio-economic status based on "head of family's" occupation.

### 5. April 28 - May 9, 1969

The "Test of Conservation of Numerousness" was administered to all control and experimental children. Testing dates were randomly assigned.

### 6. Summer and Fall, 1969

Data was tabulated and subjected to computer analysis.

### 7. Summer and Fall, 1969

Results were analyzed and written up in report form.

## III. Subjects

A total of 484 kindergarten students in the city of Denver, Colorado were the subjects in this study. They ranged in age from 65 months to 89 months with a mean age of 73.47 months. These subjects were members of eighteen classrooms picked at random from six geographic areas of the city. The choice of A.M. or P.M. classes was also randomly assigned. The teachers in these classrooms ranged in age from 22 years to 61 years with a mean age of 35.09. Their



years of teaching experience ranged from one year to 31 years with a mean of 9.739 years of experience. Their college training is summarized in the table below.

Level of College Training

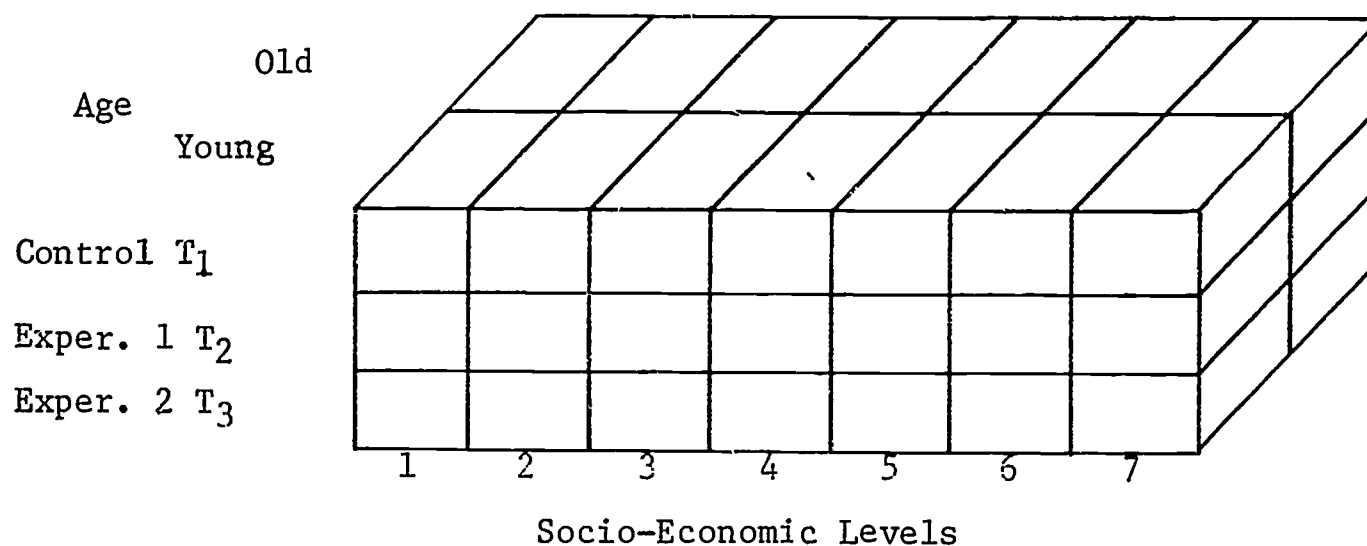
1. Less than B.A.	0
2. B.A.	13
3. B.A. plus x hours	2
4. M.A.	3
5. M.A. plus x hours	0

The following table summarizes the number of pupils in the various schools and treatment groups:

Number of Subjects in Treatment and Control Classes by School								
School	AM	PM	Control (Treatment 1)		Experimental 1 (Treatment 2)		Experimental 2 (Treatment 3)	
			Boys	Girls	Boys	Girls	Boys	Girls
Asbury	X						13	20
Ashland		X	15	11				
Ashley	X				11	16		
Ashley	X						14	10
College								
View	X		20	11				
Columbian	X				10	12		
Columbian		X					8	14
Crofton		X			6	12		
Force		X			14	13		
Godsman	X						13	14
Greenlee	X				12	11		
Harrington		X	14	16				
McMeen	X		15	15				
Mitchell	X						12	15
Philips	X		17	18				
Sherman		X	13	11				
University								
Park		X					16	16
Washington								
Park		X			15	11		
Totals	9	9	94	82	68	75	76	89

## Experimental Design

The Experimental design was a 3x7x2 factorial design with one covariate. The factors were treatments (T), socio-economic status of head of family (SES), and Age (A) (above and below the mean of 73.47 months). The dependent measure was the students' scores on the "Test of Conservation of Numerousness," Form III. The covariate was the teacher's years of teaching experience. All of the factors were considered to be fixed factors. The BMD05V was employed to analyze the data.



No attempt was made to control sex as a factor in this study since it had not proven to be a significant factor in the previous studies conducted by the author and Steffe. The children's ages were ordered from youngest to oldest and then split at the halfway point to give the classifications of "young" and "old". Piaget's studies consider age as an important factor in conservation, thus it was employed in the analysis here.

In a preliminary analysis, the teachers' years of teaching experience had some relationship and it was therefore used as a covariate in the final analysis of the data. The data were then analyzed by the analysis of covariance on the CDC 6400 with program BMD05V Biomedical Computer Program 05V.

Correlation Matrix of Variables

Pupil Variables	Sex	Age	SES	TA	TE	LT	TC
Sex	1.00	.13	.08	.29	.34	.34	.02
Age		1.00	.04	.12	.11	.09	.04
Socio-Econ. Status			1.00	.31	.30	.17	.27
Teacher's Age				1.00	.90	.28	.23
Teaching Experience					1.00	.38	.25
Level of Training						1.00	.16
Test of Conservation							1.00

#### IV. Results

Campbell and Stanley's (11, p. 8) design 6, "The Posttest Only Control Group Design" (R X <sub>1</sub>0) was employed in this study.

(R X <sub>2</sub>0)  
(R 0)

Within the strata, classrooms were randomly assigned to experimental and control groups. The treatment was given to the experimental groups and a test was administered to the subjects in experimental and control groups at the end of the experiment.

The testing instrument, "Test of Conservation of Numerousness," Form III was correlated with Form I of the same test. A Pearson Product Moment parallel forms reliability coefficient of .70 was obtained. The investigator felt that this reliability was substantial and that this form of the test would be easier to administer to the large population involved in this study than would the original Form I.

Item Correlations with Total Test Scores

Item	Form III with Form I	Item	Form III with Form I
1	.61	1	.70
2	.73	2	.78
3	.29	3	.37
4	.72	4	.65
5	.58	5	.74
6	.87	6	.84
7	.60	7	.53
8	.78	8	.55
9	.24	9	.20
10	.20	10	.44
11	.76	11	.64
12	.82	12	.74
13	.17	13	.28
14	.30	14	.82
15	.74	15	.58
16	.64	16	.78

N = 55

Age	SES	Treat	Adjusted Means	N	Standard Deviations	Covariate Means
Y	1	C	10.341	17	3.913	7.176
Y	1	E1	9.895	7	4.309	13.286
Y	1	E2	8.163	13	3.776	13.846
Y	2	C	9.806	4	2.160	11.500
Y	2	E1	10.938	6	1.862	13.333
Y	2	E2	6.761	3	3.464	21.000
Y	3	C	6.847	9	5.094	9.111
Y	3	E1	5.829	10	4.508	12.200
Y	3	E2	7.713	16	3.931	8.938
Y	4	C	7.071	16	4.270	6.250
Y	4	E1	7.048	22	4.035	11.364
Y	4	E2	6.579	25	3.341	12.840
Y	5	C	5.020	8	3.462	3.875
Y	5	E1	6.681	7	4.820	6.143
Y	5	E2	5.658	7	3.891	14.143
Y	6	C	5.386	14	3.118	4.286
Y	6	E1	5.209	12	4.144	7.083
Y	6	E2	5.980	18	4.015	6.889
Y	7	C	3.757	7	1.732	2.857
Y	7	E1	4.057	12	3.284	3.167
Y	7	E2	6.089	9	3.167	4.889
O	1	C	7.663	15	4.301	6.133
O	1	E1	7.158	7	4.353	21.286
O	1	E2	6.459	19	4.559	17.526
O	2	C	9.327	5	4.393	10.400
O	2	E1	7.708	8	2.712	23.750
O	2	E2	5.754	7	2.870	14.571
O	3	C	8.590	13	3.662	5.077
O	3	E1	8.820	12	3.233	15.917
O	3	E2	4.674	15	4.530	15.733
O	4	C	6.366	15	4.912	5.200
O	4	E1	8.288	12	4.210	11.667
O	4	E2	5.355	11	3.830	19.727
O	5	C	6.644	10	3.910	5.700
O	5	E1	9.268	11	3.668	4.000
O	5	E2	4.657	3	4.726	28.000
O	6	C	6.838	26	4.681	5.269
O	6	E1	8.742	10	3.590	3.000
O	6	E2	6.244	12	3.957	14.333
O	7	C	5.571	16	4.674	5.688
O	7	E1	6.613	7	4.158	1.571
O	7	E2	8.005	7	2.498	12.286

ANCOVA Data Table

# ANALYSIS OF VARIANCE RESULTS TABLE 1 \*\*\*

Effect	Degrees of Freedom	MS	F-value
Treatment (T)	2		3.8619 *
SES (S)	6		3.04193 **
Age (A)	1		.21105
TXS	12		1.0979
SXA	6		2.6893 *
TXA	2		2.2047
TXSXA	12		.46653
Mean Square Error	440	15.1306	

\* P < .05

\*\* P < .01

\*\*\* Only mean square error, degrees of freedom, and F-values for effects are given. All other values can be reconstructed from these.

## POST-HOC COMPARISON RESULTS

### Tukey Test

#### 1) Control - E1 - E2

Error Term:  $\sqrt{MSe \text{ (effective/ - )}} = .30656$  ( $\bar{N}$  = harmonic mean)

Means: C (n = 175)    E-1 (n = 143)    E-2 (n = 165)  
           7.0496            7.4045            6.3831

Differences:    E1 - C = .3549  
                   C - E2 = .6665  
                   E1 - E2 = 1.0214    p < .05)

#### 2) Control - E1 - E2 for lower SES (i.e. groups 5 - 6 - 7 combined)

Error term: .2778

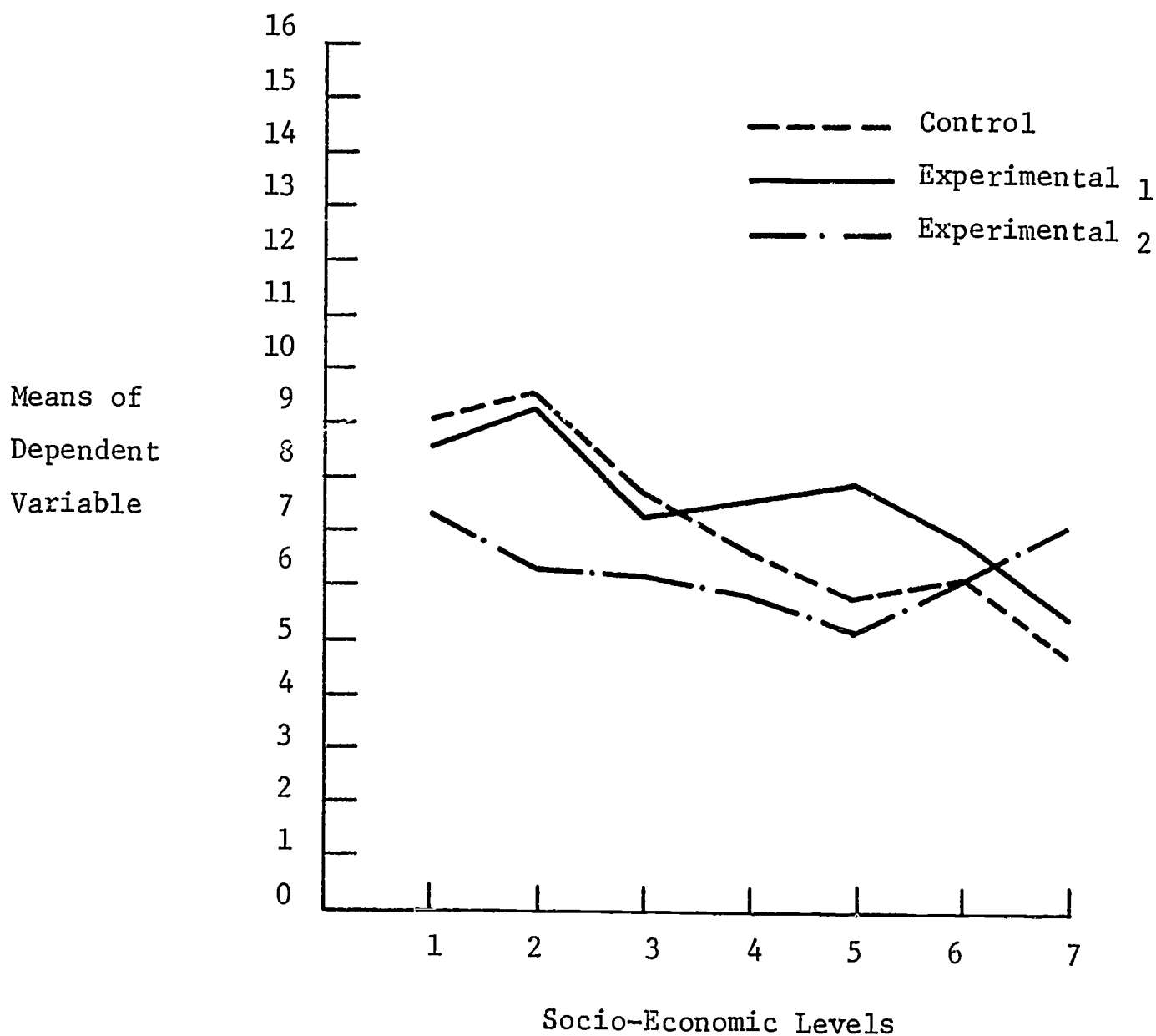
Means: C (n = 81)    E1 (n = 59)    E2 (n = 56)  
           5.8671            6.6714            6.1964

Differences    E1 - C = .8043    (p < .10)  
                   E1 - E2 = .4750  
                   E2 - C = .3293

The overall results of the analysis of variance indicate differences in the treatment at the .05 level of confidence. The adjusted mean for control was 7.0496, for Experimental 1 was 7.4045, and for Experimental 2 was 6.3831. With 2 and 440 degrees of freedom this yields an F value of 3.8619. Employing the Tukey Test, the post hoc comparison of means (after making adjustment in MSe necessary with analysis of covariance) indicated that the contributing effect was due to the difference between Experimental 1 and Experimental 2. The comparison indicated that Experimental 1 was more effective than Experimental 2 at the .05 level of confidence.

Socio-economic Status and SES by Age were also significant at the .01 and .05 levels respectively. Post hoc comparisons indicate that children coming from homes where the "head of family" was employed in the three lowest categories, performed better in the Experimental 1 teaching-learning situation than in either of the other two treatments. The interaction between Socio-Economic-Status and Age was disregarded as it did not involve an interaction with the treatment.

Graph of Means of Treatment Groups and Socio-Economic Levels of Students





Further analysis was done employing a nested design where students were nested within teachers and teachers were nested within treatment. The results indicate that there is a large variability among teachers and the variation tends to reduce differences between treatments.

ANALYSIS OF VARIANCE RESULTS, TABLE 2

Effect	Degrees of Freedom	MS	F-Value
Method	2	16.787	.236
Teachers (Method)	15	71.065	4.832*
Student (Teachers & Method)	306	14.707	

\*  $p < .05$

#### V. Conclusions

There were four questions this study was designed to answer. They are listed on page 2. The only question which would be answered affirmatively would be question number two "Is the treatment of greater value for pupils from disadvantaged backgrounds?" This is true if by combining categories 5,6 and 7 one considers these groups to be lower socio-economic groups of disadvantaged backgrounds. Then the Experimental 1 treatment, where the teachers met weekly with the investigator for in-service instruction on the use of the lessons, proved most successful of the three treatments for enhancing conservation of numerosness with children from categories, 5, 6, and 7. This approached significance at the .10 level of confidence when the Tukey Test was employed.

The interaction between Treatment and Age approaches significance with an F value of 2.2047 with 6 and 440 degrees of freedom. On closer observation, however, there is no consistent pattern among means.

#### VI. Recommendations

The results of the study seem to indicate that the "Lessons on Conservation of Numerousness" may have some value for use with children from lower socio-economic-status groups if they are conducted under the direction of a mathematics education specialist. However, further study of the learning patterns of children from categories 5, 6, and 7 seem justified before extensive changes in curricular offerings are made.

The Denver Public Schools have been using the lessons during the 1969-70 school year with children from the lower SES areas as an introduction to first grade arithmetic. The teachers reactions seem favorable for the use of the lessons for this purpose. Further analysis would be necessary to determine the appropriate groups and times for their optimum use.

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# APPENDIX 1

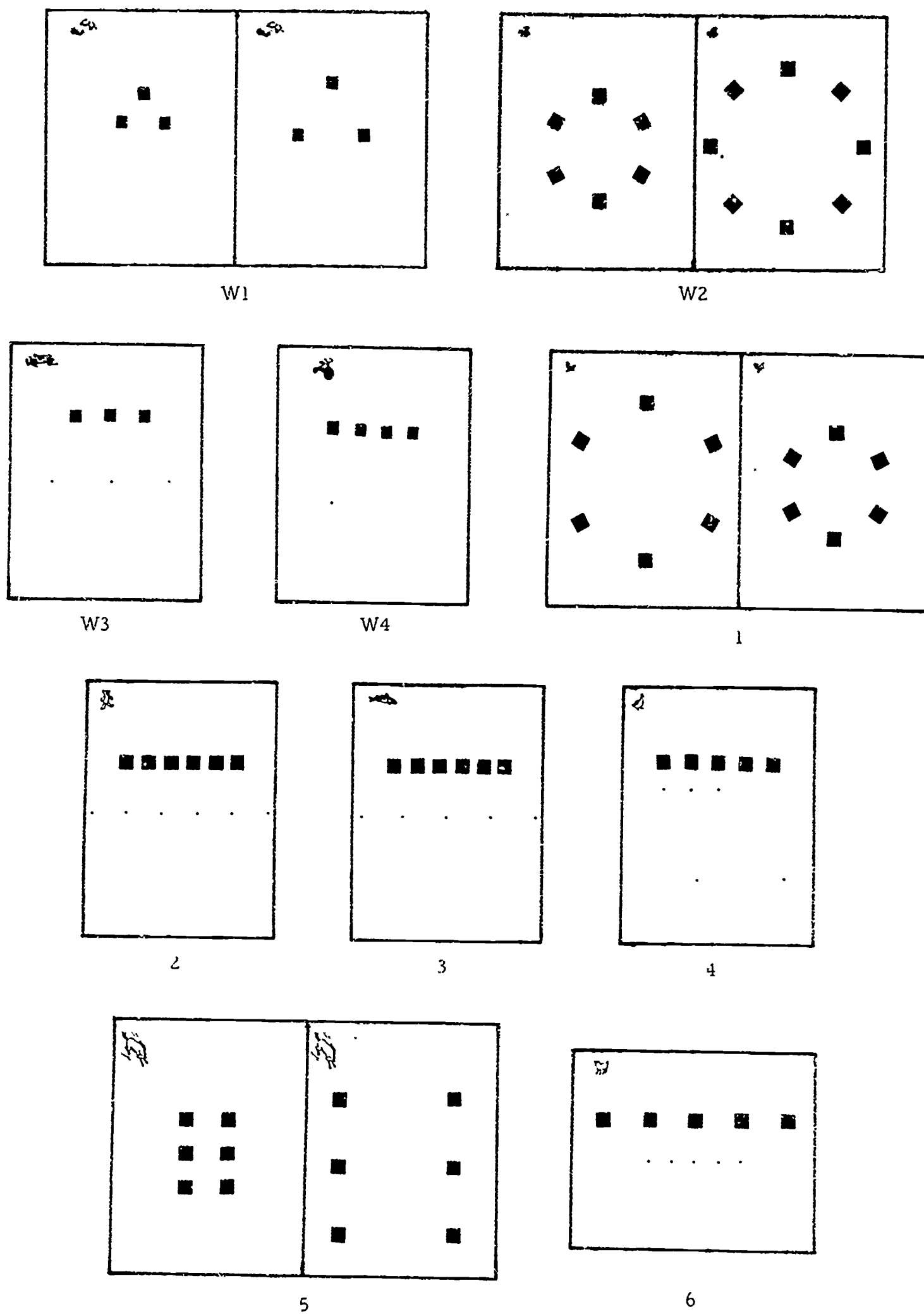
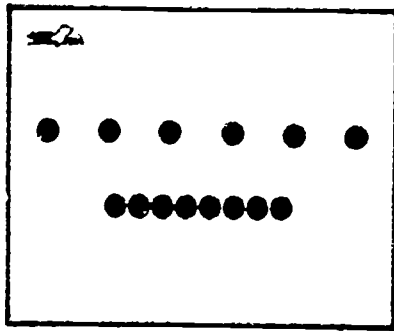
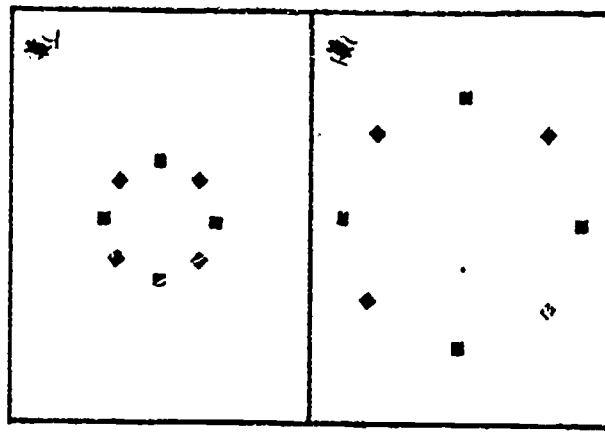


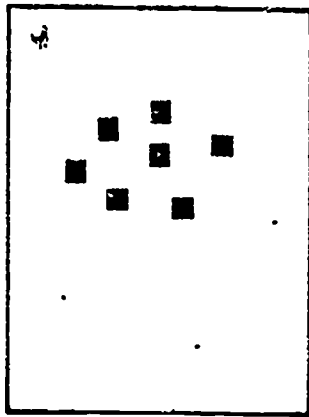
Figure 4. Test of Conservation of Numerousness



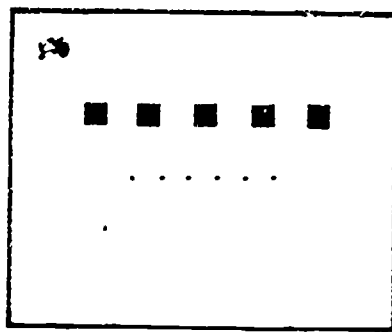
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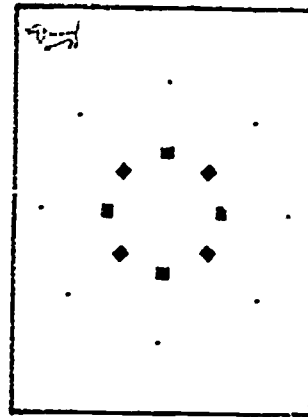
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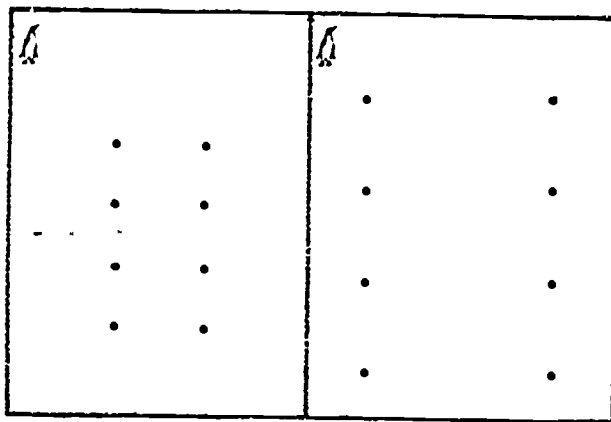
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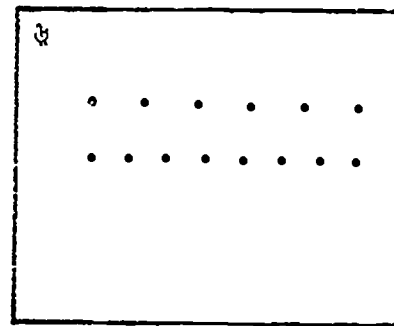
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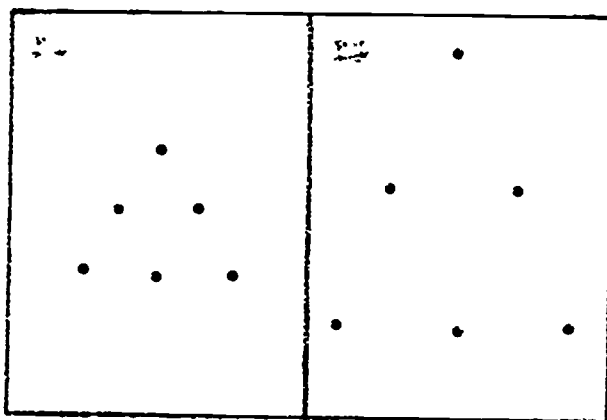
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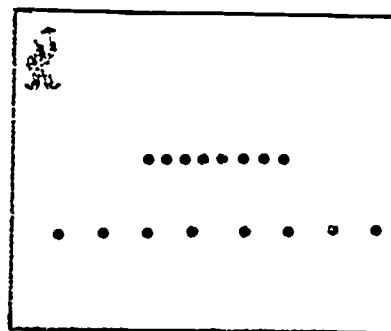
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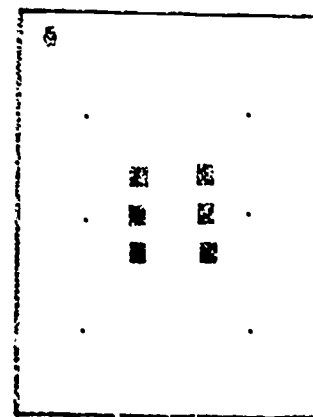
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Figure 4 (continued)



## APPENDIX 2

